# Hair Animation with Collision Detection

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## ABSTRACT

We propose an efficient method for hair animation. The motion of hairs is modeled by a simplified physical simulation. Collision between hair and a human body or other objects is used to make a realistic hair animation. Fast collision detection is required because the number of hairs is very large. Therefore, a cylindrical representation of human body or other objects is created for efficient collision detection. Because of the cylindrical representation, collision is detected with table look-up and interpolation, and computation complexity is independent of the complexity of the body. Reaction constraint method is applied for the collision response to simulate inelastic contact. This method is successfully applied to generate hair animation.

**Keywords** : human character animation, hair, collision detection.

## 1. INTRODUCTION

Human character animation is one of the most challenging topics in computer graphics (Magnenat-Thalmann and Thalmann 1991). Human hair is important to generate natural human characters. However, there remain many problems to overcome for hair animation, because the number of hairs is very large (hundreds of thousands of strands).

Several techniques have been proposed relating to hair rendering. Miller (1988) successfully generated furry object by modeling each strand as long triangles. Kajiya and Kay (1988) used three dimensional texture mapping, *texel*, to generate images of teddy bear. LeBlanc et al. (1992) used antialiasing and shadow buffer techniques to generate naturalistic images of human hair.  Rosenblum et al. (1992) also used similar techniques.  Watanabe and Suenaga (1992) proposed a 'trigonal prism and wisp model' to represent human hair.

These successful results of hair rendering have shown that more realistic human characters can be synthesized with modeling and animation techniques of hair.  However, research of modeling and animation techniques is still an open field.

The movement of hair is governed by physical law, so physically based modeling approach (Terzopoulos and Fleischer 1988) may make good results.  Animating single strand is not difficult problem using physically based approach.  However, the number of hair makes it impossible or impractical to apply precise physical simulation to the movement of hair. Therefore, methods to animate hair assuming some simplification have been proposed. Rosenblum et al. (1992) proposed an animation method of hair using a mass spring model. Anjyo et al. (1992) have proposed methods using one dimensional projective differential equations and pseudo-force field.  Both methods neglect the effect of collision between hairs for simplicity.  The collision of hair with other objects, such as the head, is simplified using a sphere or ellipsoid.  The problem of self collision of hair is very difficult.  However, the collision between hair and other objects is required to generate naturalistic movement of long hair.

Collision detection is an important issue in physically based approach, and several methods have been proposed (Platt and Barr 1988, Moore and Wilhelm 1988, Lafleur et al. 1991). Again, the number of hairs makes it impractical to apply these methods precisely.  Ordinary methods of collision detection by polygon and line intersection algorithms are too expensive to generate hair animation within the limitation of current computer technology.  A simplified approach to collision detection is also required as is the physical simulation of hair.

This paper presents a simplified and efficient method for hair animation with collision detection. Section 2 describes a dynamic model of hair based on simple differential equations of one-dimensional angular momenta. Section 3 introduces an efficient collision detection method using a cylindrical representation. Section 4 shows several examples of hair animation. Section 5 discusses advantage and limitation of the method.

## 2. DYNAMIC MODEL OF HAIR

We follow the dynamic model of hair proposed in (Anjyo et al. 1992). Although, a hinge effect is added for representing wavy hair, and a damping effect is added for more naturalistic movement. A strand of hair is modeled as a series of line segments (Figure 1). In this figure, $s_i$ $(1 \le i \le k)$ is segment, $P_i$ $(0 \le i \le k)$ is node, $d$ is the length of each segment, and $k$ is the number of segment in each strand. If the strand is not stretched, the shape of the hair is represented by the angles between segments. Taking the polar coordinate system as shown in Figure 2, the behavior of the zenith angle $\theta_i$ and the azimuth $\varphi_i$ of segment $s_i$ are observed. This polar coordinate system is defined according to the coordinate system of the parent segment.

The variables $\theta_i(t)$ and $\varphi_i(t)$ with the time parameter $t$ are governed by the ordinary differential equations:

$$I_i \frac{d^2\theta_i}{dt^2} + \gamma_i \frac{d\theta_i}{dt} = M_\theta$$

$$I_i \frac{d^2\varphi_i}{dt^2} + \gamma_i \frac{d\varphi_i}{dt} = M_\varphi, \qquad (1)$$

where $I_i$ is the moment of inertia of the segment $s_i$, $\gamma_i$ is the damping coefficient, $M_\theta$ and $M_\varphi$ are the torque according to θ and φ component respectively.

The inertial moment of segment $s_i$ is defined as

$$I_i = \frac{1}{3\,i}\lambda k^4 d^3, \qquad (2)$$

where λ is the line density. The torque $M_\theta$ and $M_\varphi$ applied to the segment $s_i$ are derived from the hinge effect $M_{\theta spring}$, $M_{\varphi spring}$ between two segments, and external moment $M_{\theta external}$, $M_{\varphi external}$ from external force, such as gravity, inertial force and wind:

$$\begin{aligned} M_\theta &= M_{\theta\,spring} + M_{\theta\,external} \\ M_\varphi &= M_{\varphi\,spring} + M_{\varphi\,external} \end{aligned} \quad . \qquad (3)$$

$M_{\theta spring}$ and $M_{\varphi spring}$ are defined as

$$\begin{aligned} M_{\theta\,spring} &= -k_\theta(\theta - \theta_0) \\ M_{\varphi\,spring} &= -k_\varphi(\varphi - \varphi_0) \end{aligned}, \qquad (4)$$

where $k_\theta$ and $k_\varphi$ are spring constants, and $\theta_0$ and $\varphi_0$ are the initial angles.

External moments are defined as

$$\begin{aligned} M_{\theta\,external} &= u\,F_\theta \\ M_{\varphi\,external} &= v\,F_\varphi \end{aligned}, \qquad (5)$$

where $u$ is (1/2)$d$, $v$ is the half length of the segment that is the projection of $s_i$ onto the φ plane (see Figure 3 and 4). $F_\theta$, $F_\varphi$ are the "θ, φ-component" of force **F** respectively.

The above $\theta$ component $F_\theta$ of the applied force **F** is the scalar value defined by $F_\theta = (\mathbf{F}, \mathbf{V}_\theta)$, where $\mathbf{V}_\theta$ is the unit vector on the $\theta$ plane that is perpendicular to the segment $s_i$. Similarly, the $\varphi$ component $F_\varphi$ is defined by $F_\varphi = (\mathbf{F}, \mathbf{V}_\varphi)$, where $\mathbf{V}_\varphi$ is the unit vector on the $\varphi$ plane that is perpendicular to the projected segment of $s_i$ onto the $\varphi$ plane.

The external force **F** is defined as

$$F = \rho d\ (g + a) + d\ f, \tag{6}$$

where **g** is the acceleration due to gravity, **a** is the acceleration due to the movement of the head itself, and **f** is the density of the applied force, such as wind.

In the numerical simulation, equation (1) is discritized as

$$\begin{aligned} \theta_i^{n+1} - 2\theta_i^{n} + \theta_i^{n-1} + \gamma_i \Delta t\,(\theta_i^{n} - \theta_i^{n-1}) &= (\Delta t)^2 M_\theta \\ \varphi_i^{n+1} - 2\varphi_i^{n} + \varphi_i^{n-1} + \gamma_i \Delta t\,(\varphi_i^{n} - \varphi_i^{n-1}) &= (\Delta t)^2 M_\varphi\ . \end{aligned} \tag{7}$$

The calculation starts with the segment $s_1$, and the new angle of $s_i$ is successively determined using (7).

## 3. COLLISION DETECTION AND RESPONSE

In order to generate naturalistic hair animation, collision between hair and other objects must be considered. Ordinary collision detection methods find intersections between polygons and points. However, this method is impractical because of the number of hairs, and the number of polygons of human body or other objects. It is desirable to apply an efficient collision detection method whose complexity is independent of the complexity of objects. In this section, we propose a simplified and efficient collision detection method for hair animation. This method utilizes a cylindrical representation.

### 3.1 Collision Detection using Cylindrical Projection

Collision detection in our case is to determine whether strand segment is inside of other objects or not. A correct test must consider line segments and objects. However, we consider only node point and objects test for simplicity. Then the problem is to determine whether node **P** is inside of the objects or not.

First, we create a cylindrical representation of human body or other objects. The human body is very irregular and cannot be described as one cylindrical representation. Therefore, we divide human body or other objects into several parts, and each part is described with cylindrical representation. For example, the human body is divided into a head, trunk, arms, and legs. Each part is described in a cylindrical coordinate system as $(r, \theta, y)$, where $r$ is radius, $\theta$ is azimuth, and $y$ is height. We prepare an array of radius on discretized azimuth $i\Delta\theta$ and height $j\Delta y$ $(1 \le i \le m,\ 1 \le j \le n)$ by calculating intersection between line and object (Figure 5). Along with the radius, we prepare the normal vector on each sample point for calculating collision response. Figure 6 shows examples of a cylindrical representation of the human body. Figure 6a is the original human body model with 7000 polygons. Figure 6b shows head and trunk model represented by cylindrical coordinate system. These two models are defined with [40 x 40] array of radius values.

When a cylindrical representation is created, collision is easily detected. Suppose a node point **P** is tested whether it is inside of the object of not. First, we translate point **P** into cylindrical coordinates as $(r_p, \theta_p, y_p)$. Second, we obtain the corresponding point **Q** of the object with the same azimuth $\theta_p$ and height $y_p$ as point **P**. If we assume the smoothness of object surface, position of **Q** can be approximated by linear interpolation as follows. We find index $(i, j)$ which satisfies

$$
\begin{aligned}
&i\,\Delta\theta \le \theta_p < (i+1)\Delta\theta \\
&j\,\Delta y \le y_p < (j+1)\Delta y .
\end{aligned}
\qquad (8)
$$

Radius of point **Q** is then approximated as:

$$
\begin{aligned}
r_p = {} & r(i,j)(1-s)(1-t) + r(i+1,j)s(1-t) \\
& + r(i,j+1)(1-s)t + r(i+1,j+1)s\,t ,
\end{aligned}
\qquad (9)
$$

where

$$
s = \frac{\theta_p - i\,\Delta\theta}{\Delta\theta}
$$

$$
t = \frac{y_p - j\,\Delta y}{\Delta y} . \qquad (10)
$$

Collision detection is now reduce to comparing the radius part of point **P** and that of point **Q**. If $r_q$ is greater than $r_p$ then point **P** is inside the object. Otherwise point **P** is outside of the object.

When point **P** is inside of the object, point **T** on the surface of object that is nearest from **P** is also required for collision response. Point **T** is approximated as follows (Figure 7).

1. Normal vector **N** at point **Q** is the obtained using bi-linear interpolation. We use this normal vector as an approximation of the normal vector at point **T**.

2. Point **T** is on the surface of object and it is nearest to point **P**. Thus, vector **PT** is parallel to normal vector **N**. Then point **T** is approximated as

$$\mathbf{PT} = (\mathbf{PQ} \bullet \mathbf{N})\,\mathbf{N}\ . \qquad (11)$$

The proposed collision detection requires only table look-up and bi-linear interpolation. Therefore, it is very efficient and the computational cost is independent of the complexity of the object.

### 3.2 Collision Response using Reaction Constraint

If a hair strand is inside of the body, the reaction constraint method (Platt and Barr 1988) is applied to keep hair outside of the body. Let $\mathbf{F}_{input}$ be the applied force to node point **P**. Then unconstrained component of $\mathbf{F}_{input}$ is

$$\mathbf{F}_{unconstrained} = \mathbf{F}_{input} - (\mathbf{F}_{input} \bullet \mathbf{N})\,\mathbf{N}, \qquad (12)$$

where **N** is the normal vector at point **T** (see Figure 8). This force $\mathbf{F}_{unconstrained}$ has no relationship with the collision and is not changed.

The constrained force to avoid the collision is

$$\mathbf{F}_{constrained} = -\,(k\,\mathbf{PT} + c\,\mathbf{V} \bullet \mathbf{N})\,\mathbf{N}, \qquad (13)$$

where $\mathbf{V}$ is the velocity of point $\mathbf{P}$, $\mathbf{T}$ is the nearest point on the surface from point $\mathbf{P}$, and $k$ is the strength of the constraint and $c$ is the damping coefficient. Critical damped motion is obtained by letting $c = \sqrt{2k}$.

The output force which is applied to point $\mathbf{P}$ is a summation of $\mathbf{F}_{unconstrained}$ and $\mathbf{F}_{constrained}$.

$$\mathbf{F}_{output} = \mathbf{F}_{unconstrained} + \mathbf{F}_{constrained} . \tag{14}$$

Using equation (12), (13) and (14), the output force is written as

$$\mathbf{F}_{output} = \mathbf{F}_{input} - (\mathbf{F}_{input} \bullet \mathbf{N})\,\mathbf{N} - (k\,\mathbf{PT} + c\,\mathbf{V} \bullet \mathbf{N})\,\mathbf{N}. \tag{15}$$

To simulate inelastic collision, we apply reaction constraint only if the input force is not lifting the point $\mathbf{P}$ away from the surface.

## 4. RESULTS

To demonstrate the dynamic simulation, several animation sequences of hair movement have been created. Figure 9 shows the initial style of hair. The number of strands is about 1000, and each strand consists of about 40 segments.

The proposed method is efficient to make hair animation because we simplify the physical simulation and collision response. However, it is expensive to simulate all hundreds thousand strands of hair. When we observe real hairs, many neighboring strands move similarly. Thus, if we simulate the movement of a cluster of strands not that of each strand, it reduces the computation time significantly.

The cluster model of strands we have applied is very simple one. We simulate the movement of typical strands and generate other strands by adding random numbers to the origin of the typical strands. Figure 10 shows the hair model generated by adding 20 strands for each typical strand of initial hair style, so the total number of strands is 20,000.

Figure 11 shows four frames taken from a short film representing a windy scene. In this scene, only the force of wind and gravity are applied. The wind is coming from the front of the face. Note that strands do not penetrate the head or shoulders because of the collision response.

Figure 12 shows five frames taken from the head shaking scene, where the head move left, right, and finally left. A little wind is also applied coming from the front of the face.

Figure 13 illustrates the image rendered using a pixel blending and shadow buffer technique (LeBlanc et al. 1992). The number of strands in this image is about 40,000.

An animation software based on the presented method has been implemented on a Silicon Graphics Iris Power Series workstation. The wall clock time is typically 30 seconds per frame with collision response. The clock time for the collision response alone is 3 seconds. This result shows the efficiency of the proposed collision detection method.

## 5. DISCUSSIONS

Ordinary collision detection methods calculate intersection between objects. Therefore, the complexity of algorithm is $O(n^2)$ where $n$ is the number of polygons. Space subdivision techniques, such as octree and voxel reduce the computation cost. However, preparing these data structures is expensive when the object moves or deforms. The complexity of the proposed method is $O(n)$ because of the table look-up in cylindrical coordinate system. In addition, preparing the cylindrical representation is not expensive. This is the advantage of the proposed method.

This collision detection method is a rough approximation of precise one, such as line and triangle intersection checking. Therefore, it sometimes fails to detect collisions because the resolution of the table is not enough, or because the cylindrical representation is not sufficient to describe the objects. For example, shoulders and top of head are not described precisely with cylindrical representation. However, experiments show that this approximation can be sufficient for the purpose of animating hair.

Though this method utilizes a cylindrical coordinate system, other coordinate systems can be used, such as a spherical coordinate system. The only requirement is the mapping from three-dimensional space to two-dimensional space.

The proposed technique is simple and efficient for collision detection. Therefore, it can be applied to other applications as a rough approximation or efficient culling method for a more precise collision detection algorithm.

This method cannot be applied to the problem of self collision between the hairs because it is difficult to represent the hair itself in cylindrical coordinate system. The self collision is required for the more realistic hair movement. Therefore, it must be detected with some other techniques.

## 6. CONCLUSIONS

We have presented an efficient method for animating hair. A simplified physical simulation and collision detection method make it possible to generate naturalistic motion of hair with reasonable computation cost. The collision detection algorithm for the hair and other objects is efficient and the computation time is independent of the complexity of object.

The proposed method neglects strand-strand interaction for simplicity. However, realistic hair motion cannot be achieved without this effect, especially with a voluminous hair style. Precise treatment of strand-strand interaction will be too expensive within the limitation of current computational power. Therefore, some simplification of strand-strand interaction must be developed.

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2. moment etc.
3. Introduction, rendering

Several techniques have been proposed relating to hair rendering. Rendering methods of furry objects, such as teddy bear (Kajiya and Key 89) or spider (Miller 88) have been proposed. More recent works (LeBlanc 92, Rosenblum 92, Watanabe 92) have shown that naturalistic human hair rendering can be performed as variations of previous research, including above techniques. These successful results have led us to achieve more realistic syntheses of human characters, along with the development of modeling and animation techniques. However research of modeling and animation techniques is not completed yet.

Recently, Shinya and Forgue (1992) proposed collision detection method using rasterization This method rasterizes the projection of objects and calculates *z*-values. Collision is detected by sorting *z*-list. Though this method for collision detection is similar to the method presented in this paper, it is based on orthogonal projection or perspective projection. This can be drawback for some application such as hair animation. In contrast, the method proposed in this paper requires merely mapping from two-dimension to three-dimension. Cylindrical projection is efficient mapping for hair animation.

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